UNIVERSITY OF TORONTO
FACULTY OF ARTS & SCIENCE | Department of Mathematics

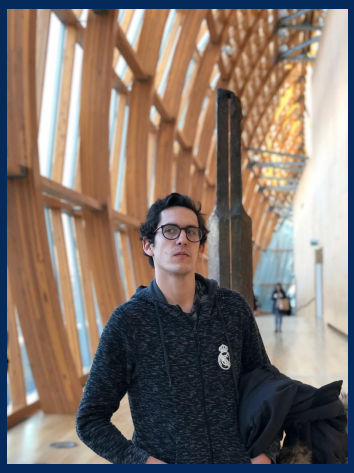
Departmental PhD Thesis Exam

Wednesday, June 12 at 2:00 p.m (sharp)
BA6183/via Zoom (hybrid)

PhD Candidate : Joaquín Sánchez García

Supervisor : Dr. Robert McCann

Thesis title : 4 Problems in optimal transportation



Abstract

This thesis is devoted to the study of 4 different problems for which we use the theory of optimal transportation: 1. The generalization of the Schrödinger Problem to synthetic Lorentzian geometries. 2. The small time existence for solutions for the aggregation equation on compact Riemannian manifolds for non-regular interaction potentials via the minimizing movement scheme. 3. The technique of measure pre-conditioning general Machine-Learning tasks and Domain Adaptation transfer learning. 4.The generalization of an economic model of Roy for partition of labor including occupational choice as a constraint.

1 Part 1: The Schrödinger problem in synthetic Lorentzian geometries

The Schrödinger problem refers to the minimization of relative entropy with respect to a reference measure. The Schrödinger problem is usually analyzed in two related formulations: the static and the dynamic Schrödinger problems. We study both approaches. One of the main questions of the Schrödinger problem is whether or not the solutions to the entropically regularized optimal transport problem converge to solutions of the optimal transport problem. In the dynamical setting, this property amounts to study the Large Deviation Principles of the reference measure.

We study a Levy-like construction which emulates the behaviour of Brownian bridges which allows us to recover a partial version of the entropic convergence in the non-smooth Lorentzian case.

2 Part 2: The aggregation equation via the minimizing movement scheme in compact Riemannian manifolds

We study the small-time existence of solutions for non-smooth potentials via the minimizing movement scheme. The theory of gradient flows in metric spaces does not consider a potential non-regularity of potentials in the cut-locus. The presence of the cut-locus presents a difficulty for the JKO scheme to choose a direction, nevertheless we show explicitly a time bound for which we can flow the minimizing movement scheme.

3 Part 3: Measure Pre-conditioning in Machine-Learning

We study a new technique to improve convergence of algorithms for specific ML-tasks. We show that if the modifications of the problem at level n (sample size) are done in a specific way (full learner recovery systems) we can show analytical subsequential convergence to the original model. This technique seems to be specifically important for Domain Adaptation in transfer learning.

4 Part 4: Generalizing an economic model of Roy for labor partition using occupational choice as a constraint

We study the analytical properties of a generalization of the economic model for labor force partition studied by Dr. Roy. The new model proposed by Dr. Siow, includes occupational choice as a constraint rather than a consequence. This difference allows us to rewrite the problem in an analytically useful way.