UNIVERSITY OF TORONTO
FACULTY OF ARTS & SCIENCE | Department of Mathematics

Departmental PhD Thesis Exam

Tuesday, July 9th, 2024 at 11:00 a.m. (sharp)
via Zoom / BA6183

PhD Candidate : Amirmasoud Geevechi

Supervisor : Robert Jerrard

Thesis title : A Gluing Problem for a Gauged Hyperbolic PDE

Abstract

In this thesis, we study the dynamic Abelian Higgs model in dimension 3 at the critical coupling. This is a system of partial differential equations which enjoys local symmetries known as gauge transformations. The stationary finite energy solutions to these equations in dimension 2 have been classified by Jaffe and Taubes in 1980, the so called vortex configurations. In 1992, Stuart has proved that one can construct solutions near the critical coupling regime in dimension $1+2$ whose dynamics are approximated by a finite dimension Hamiltonian system on the moduli space which reduces to the geodesic flow at the critical coupling.

In this project, we study how one can glue the vortex configurations to find dynamic solutions in dimension 3. More precisely, we prove that one can construct solutions which are approximated by wave maps to the moduli space of vortex configurations. The proof involves an ansatz to construct approximate solutions and then add perturbations. In the ansatz, we go through an iterative mechanism to reduce the error of the approximate solution so that it is prepared to be perturbed to find an honest solution.

In both steps of the project, the ansatz and perturbation, the choice of gauge is crucial. It is noteworthy that the choices of gauge are different in these two steps. We proceed by a choice for gauge, simplify the equations and then we have to decompose the quantities into two components, the zero modes (the tangent vectors to the moduli space) and the orthogonal complement to zero modes. According to the Higgs mechanism, stability is available for the components orthogonal to zero modes. In this regard, in the ansatz, the dynamics of zero modes is designed in such a way that the orthogonality condition to zero modes is satisfied. In the perturbation part, the dynamics of zero modes is forced by the evolution of orthogonal components. Obtaining desired estimates for the tangential part requires taking advantage of explicit structure of equations, rather than the usual estimates. Also, the number of iterations in the ansatz should be high enough so that the desired estimates hold for the tangential part.