UNIVERSITY OF TORONTO
FACULTY OF ARTS & SCIENCE | Department of Mathematics

# Departmental PhD Thesis Exam

Thursday, June 27th, 2024 at 11:00 a.m. (sharp)
via Zoom / BA6183

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Supervisor : Bálint Virág

Thesis title : Directed polymers in the intermediate disorder regime and the Seppäläinen–Johansson model

## Abstract

In this thesis, we study two discrete models of random growth in the Kardar–Parisi–Zhang (KPZ) universality class: the directed polymer and the Seppäläinen–Johansson first-passage percolation model.

The directed polymer was introduced by Huse and Henley as a model for the domain wall in a ferromagnetic Ising model with random bond impurities. This model depends on a parameter $\beta$, the inverse temperature. We consider the intermediate disorder regime, which consists in taking $\beta$ to depend on the length of the polymer $2n$, with $\beta = n^{-\alpha}$ for some $\alpha > 0$. In this regime, there is a critical phase transition that happens at $\alpha = \frac{1}{4}$. When $\alpha > \frac{1}{4}$, the fluctuations of the free energy are of order $n^{(1-4\alpha)/4}$ and converge to a Gaussian. For $\alpha < \frac{1}{4}$, it was conjectured that the polymer should fall back in the KPZ regime, and that the fluctuations should instead be of order $n^{(1-4\alpha)/3}$, and converge after rescaling to the Tracy–Widom GUE distribution. We prove this conjecture for $\frac{1}{8} < \alpha < \frac{1}{4}$ for arbitrary i.i.d weights with exponential moments.

The Seppäläinen–Johansson model was introduced by Seppäläinen as a simplified version of first-passage percolation where he was able to explicitly compute the limiting shape for Bernoulli weights. The behaviour of the fluctuations for this process were later studied by Johansson. We consider a generalization of this model, involving two families of i.i.d random variables $\{\xi_{ij}\}$ and $\{\eta_{ij}\}$ corresponding to the weights of the horizontal and vertical edges respectively. We obtain an explicit formula for the limiting shape of the first-passage distance expressed in terms of the corresponding limit shapes of the two sets of weights for the Seppäläinen–Johansson model. We also study the limiting fluctuations of this model when at least one of the sets of weights is Bernoulli distributed, and we prove that these converge to the $\text{Airy}_2$ process.