Departmental PhD Thesis Exam

Wednesday, December 11, 2024 at 1:30 p.m.
via Zoom / BA6183

PhD Candidate :   Ahmed Ellithy

Supervisor :   Spyros Alexakis

Thesis title :   Existence of Static Vacuum Extensions for Near-Schwarzschild Spheres: A New Approach to the Bartnik Extension Problem

Abstract

We establish the local well-posedness of the Bartnik static metric extension problem for arbitrary Bartnik data that perturb that of any sphere in a Schwarzschild $\{t = 0\}$ slice. Our result in particular includes spheres with arbitrary small mean curvature. We introduce a new framework to this extension problem by formulating the governing equations in a geodesic gauge, which reduce to a coupled system of elliptic and transport equations. Since standard function spaces for elliptic PDEs are unsuitable for transport equations, we use certain spaces of Bochner-measurable functions traditionally used to study evolution equations. In the process, we establish existence and uniqueness results for elliptic boundary value problems in such spaces in which the elliptic equations are treated as evolutionary equations, and solvability is demonstrated using rigorous energy estimates. The precise nature of the expected difficulty of solving the Bartnik extension problem when the mean curvature is very small is identified and suitably treated in our analysis.