UNIVERSITY OF TORONTO
FACULTY OF ARTS & SCIENCE | Department of Mathematics

Departmental PhD Thesis Exam

Monday, July 29th, 2024 at 2:00 p.m. (sharp)
via Zoom / BA6183

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Supervisor : Jacob Tsimerman

Thesis title : Hyperbolicity and Rational Points on Complex Ball Quotients

Abstract

Let $X = \Gamma\backslash\mathbb{B}^n$ be an n-dimensional complex ball quotient by a torsion-free non-uniform lattice Γ whose parabolic subgroups are unipotent. Let $\overline{X}$ be the unique toroidal complication of X.

In the first part of this thesis we prove positivity properties of $\Omega^1_{\overline{X}}$ and $\Omega^1_{\overline{X}}\big(\log(D)\big)$ depending intrinsically on X. We prove that $\Omega^1_{\overline{X}}\big(\log(D)\big)\langle -rD\rangle$ is ample for all sufficiently small rational numbers $r > 0$, and $\Omega^1_{\overline{X}}\big(\log(D)\big)$ is ample modulo D. Further, we conclude that if the cusps of X have uniform depth greater than 2π, then $\Omega^1_{\overline{X}}$ is semi-ample and is ample modulo D, and all subvarieties of X are of general type.

In the second part of this thesis we prove that the volumes of subvarieties of X are controlled by the systole of X, which is the length of the shortest closed geodesic of X. There are a number of arithmetic and geometric consequences: the systole of X controls the growth rate of rational points on X, uniformly in the field of definition of $\overline{X}$. Also, we obtain effective global generation and very ampleness results for multiples of the canonical bundle $K_{\overline{X}}$. These results follow from the bound we find for the Seshadri constant of $K_{\overline{X}}$ in terms of the systole.