A nonlinear function space

Assume that Ω is an open subset of $\mathbb{R}^n$ and $u : \Omega \to \mathbb{R}$ is C^2 function. Writing D^2u to denote the matrix of second partial derivatives of u, a version of the change of variables formula for multiple integrals implies that

$$\int_{x\in\Omega} |\det D^2u| \, dx = \int_{p\in\mathbb{R}^n} \#\{x \in \Omega : Du(x) = p\} \, dp, \tag{1}$$

where for any set S,

$$\#S := \begin{cases} \text{the cardinality of } S & \text{if } S \text{ is finite} \\ +\infty & \text{otherwise.} \end{cases}$$

Equation (1) contains the assertion that the integrand on the right-hand side is integrable if the integral on the left is finite.

This project will study questions related to the goal of identifying the largest natural space of functions for which the expression on the right-hand side of (1) makes sense and is finite, and investigating properties of these functions. Work on these questions may involve elements of calculus on manifolds, real analysis, and algebraic topology.

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