UNIVERSITY OF TORONTO
FACULTY OF ARTS & SCIENCE | **Department of Mathematics**

Departmental PhD Thesis Exam

Friday, August 30, 2024 at 10:00 a.m. (sharp)
via Zoom / BA6183

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Supervisor : Professor Dror Bar-Natan

Thesis title : On the Gassner invariant of braids and string links

Abstract

In this thesis we delve into the computation of the Gassner invariant for string links, which are a more generalized form than braids, utilizing a (co)homological approach. We restrict this (co)homology invariant, denoted as $\mathcal{G}_h$, to pure braids, leading to the derivation of the Gassner representation.

We introduce the concept of "flying cars," which assigns an invariant $\mathcal{C}(L)$ to an $(n+1)$-component string link L. This invariant, an $n \times n$ matrix, has entries in the field $\mathbb{Q}(t_0, t_1, \ldots, t_n)$. We establishes a connection between the invariant $\mathcal{C}(L)$ and the homology Gassner invariant $\mathcal{G}_h(L)$ of L through the formula $\mathcal{G}_h(L) = \left(D_n \cdot \mathcal{C}(L) \cdot D_n^{-1}\right) // \rho_{col} // m^t$. Here, D_n is a diagonal matrix, m^t denotes matrix transpose, and ρ_{col} represents column permutation. We prove that $\mathcal{C}(L)$ is indeed an invariant of string links under the Reidemeister moves, thereby directly verifying the invariance of the homology Gassner invariant.

Moreover, we provide formulas for the intersection product $\mu := \langle -, - \rangle : H_1(P;\mathcal{F}) \times H_1(P;\mathcal{F}) \to \mathcal{F}$, which is defined on the cycles of the homology group $H_1(P;\mathcal{F})$. In this context, P is an $(n+1)$-punctured disk viewed as a subspace of the complement X of an $n+1$ string link, and $\mathcal{F}$ is a local coefficient system on X determined by the abelianization map $\varepsilon : \pi_1(X, x_0) \to \langle t_0, t_1, \cdots, t_n \rangle$. This map takes values in the free abelian group $\langle t_0, t_1, \cdots, t_n \rangle$. We conclude by verify that the homology Gassner invariant is unitary with respect to this intersection product.