



Departmental PhD Thesis Exam

Tuesday, September 10th, 2024 at 10:00 a.m. (sharp)
via Zoom



PhD Candidate : Virginia Pedreira

Supervisor : Balint Virág

Thesis title : Ordering of the Tracy-Widom
beta distributions and fractal dimension
of the level sets of the directed landscape
in the temporal direction

Abstract

The first part of the thesis is related to the Tracy-Widom distribution. We give a stochastic comparison and ordering of the Tracy-Widom distribution with parameter β . In particular, we show that as β grows, the Tracy-Widom random variables get smaller modulo a multiplicative coefficient.

The second part of the thesis is related to the directed landscape. The directed landscape, \mathcal{L} , is a random 'metric' on \mathbb{R}^2 that arises as the rescaled limit of last passage percolation. We show that the level sets of last passage percolation converge to the level sets of the directed landscape in the Euclidean Hausdorff metric. We also describe the fractal nature of the level sets of the directed landscape. In particular, we prove that the level sets of $\mathcal{L}(0,0;0,t)$ have Hausdorff dimension of $2/3$ with positive probability. We prove this by finding matching upper and lower bounds. We provide an upper bound for the Hausdorff dimension in the usual way: by counting the number of squares that cover the level set. In the case of the lower bound, we provide sufficient conditions on the one and two-point density of any stochastic process to obtain a lower bound of the Hausdorff dimension of its level sets. This theorem generalizes for stochastic processes whose densities are not proved to exist. In that case, the conditions are on the one and two-point probability of being ε close to the level set. Then, we prove that the directed landscape satisfies the conditions on the two-point probability mentioned above. We conclude that $2/3$ is also the lower bound of the level set of $\mathcal{L}(0,0;0,t)$ with positive probability.