UNIVERSITY OF TORONTO
FACULTY OF ARTS & SCIENCE | Department of Mathematics

Departmental PhD Thesis Exam

Wednesday, December 13, 2023 at 3:00 p.m. (sharp)
via Zoom / BA6183

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Supervisor : Adam R. Stinchcombe

Thesis title : Temporal Difference Learning for Viscous Incompressible Flow

Abstract

This thesis presents a stochastic numerical method for computing viscous incompressible flow. By Itô's lemma, the solution to a linear parabolic PDE is a martingale over an appropriate probability measure induced by Brownian motion. Given an initial boundary value problem, a functional corresponding to the martingale condition is minimized numerically through deep reinforcement learning. This methodology is well-suited for high dimensional PDEs over irregular domains, as it is mesh-free and sampling techniques can avoid the curse of dimensionality.

The extension to computing viscous incompressible flow is done by first formulating a martingale condition for the viscous Burgers' equation. Its solution is obtained by a fixed point iteration for which a proof of convergence in L^2 is provided. The constrained minimization problem subject to divergence-free vector fields is designed for the incompressible Navier-Stokes equations. The velocity is determined without the pressure gradient. The stochastic numerical method avoids difficulties arising from coupling of velocity and pressure terms by globally maintaining incompressibility. Furthermore, pressure can be recovered from the computed velocity in a post-processing step.

The numerical implementation details are provided, including errors from statistical sampling. Simulations of various flow scenarios are showcased, including those with analytical solutions such as Stokes' flow in a revolving ball, Poiseuille flow, and the Taylor-Green vortex. Additional validation is acquired from comparing against numerical solutions for cavity flow, and flow past a disk. Analysis is undertaken to determine bounds on the statistical and numerical error. A number of improvements in deep learning and generalizations to broader classes of PDES are proposed as possible avenues of future research. Software is available at `github.com/mskpark/DRLPDE`.