Problems in graph discrepancy

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This project intends to study some problems in the discrepancy of graphs. Roughly speaking, discrepancy is a measure of how "complicated" a set system is. More precisely, for a family $\mathcal{F}$ of subsets of a set X, the *discrepancy* of $\mathcal{F}$ is the maximum integer d such that in any 2-coloring of the elements of X, there is a set $A \in \mathcal{F}$ that has at least d more elements of one color than of the other. Thus, the discrepancy of $\mathcal{F}$ is the *maximum imbalance* guaranteed to exist in any 2-coloring of the ground set X.

In recent years there has been some interest in problems of this type where the ground set X is the set of **edges** of a graph G, and the set system $\mathcal{F}$ is the set of all subgraphs of G of a given type, such as perfect matchings, spanning trees, Hamilton cycles, triangle-factors, etc. A typical result in the area is as follows: If the minimum degree of an n-vertex graph G is larger than a given number, then every 2-coloring of $E(G)$ contains a given structure with large discrepancy (i.e., with one color appearing significantly more than the other). The key question is to find the *minimum degree threshold* guaranteeing this. Another direction of study is to consider the case where G is a random graph.

In this project we aim to study problems of this type. In particular, we will work to extend some known results to more than 2 colors. The notion of discrepancy generalizes naturally to more than 2 colors: The k-color discrepancy of a set system $(X, \mathcal{F})$ is the maximum d such that in every k-coloring of X, there is $A \in \mathcal{F}$ such that at least $\frac{|A|+d}{k}$ of the elements of A have the same color (so, as in the 2-color case, A is biased towards one of the colors).

One problem that we will study is the k-color discrepancy triangle-factors, and, more generally, K_r-factors. (A triangle-factor is a collection of vertex-disjoint triangles which cover all vertices.) The minimum degree threshold for having a high-discrepancy triangle-factor is known for 2 colors, and we will work to find the threshold for any number of colors.

Working on such problems is a good opportunity for students to become acquainted with advanced and sophisticated techniques in extremal combinatorics.

Some background in combinatorics, such as MAT344 and/or MAT332, is recommended.