UNIVERSITY OF TORONTO
FACULTY OF ARTS & SCIENCE | Department of Mathematics

Departmental PhD Thesis Exam

Monday, September 30th, 2024 at 1:00 p.m. (sharp)
via Zoom / BA6183

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Supervisor : Stephen Kudla

Thesis title : Symplectic Theta Functions and Theta Lifts of Modular Forms to Split Orthogonal Groups

Abstract

We define a notion of modularity for a function on the Riemannian symmetric space $\mathscr{D}_m = G/K$ for $G = O(m,m)$. We define symplectic theta functions which are functions on $\mathscr{D}_m \times \mathscr{H}_n$, where $\mathscr{H}_n$ is the Siegel upper half space of genus n, which are modular in both variables.

We pair these symplectic theta functions with modular forms on $\mathscr{H}_n$ (essentially via the Petersson product) to obtain modular forms on $\mathscr{D}_m$, and we compute their Fourier coefficients. We do this for cusp forms and weakly holomorphic modular forms in the case when $n = 1$, and for cusp forms when $n > 1$. We do this by grouping terms in a sum that gives the Fourier coefficient in terms of $Sp_n(\mathbb{Z})$ orbits. We find that only terms corresponding to nondegenerate orbits of full rank contribute to the Fourier coefficient. We also exhibit a formula directly relating lifts of a fixed cusp form f to $\mathscr{D}_m$ for $m > 2n$ to the lift to $\mathscr{D}_{2n}$.

In the case where $n = 1,2$ we obtain complete explicit formulas, and in the case where $n > 2$ we reduce the computation of the Fourier coefficients to the calculation of a single integral.