UNIVERSITY OF TORONTO
FACULTY OF ARTS & SCIENCE | Department of Mathematics

# Departmental PhD Thesis Exam

Monday, February 10th, 2025 at 10:00 a.m. (sharp)
via Zoom / 63 St. George Street, Room 111

PhD Candidate : Kyle Waller Bower

Supervisors : Adam R. Stinchcombe and Spyros Alexakis

Thesis title : The Electrostatic Problem for Piecewise Constant Conductivities in Two Dimensions: Numerical Methods and Optimal Regularity


## Abstract

We present a numerical method for solving the elliptic partial differential equation problem for the electrostatic potential with piecewise constant conductivity and a Neumann boundary condition. Our method employs an integral equation approach for which we derive a system of well-conditioned integral equations by representing the solution as a sum of single layer potentials. The fast multipole method is used to accelerate the generalized minimal residual method solution of the integral equations. For efficiency, we adapt the grid of the Nyström method based on the spectral resolution of the layer charge density. Additionally, we present a method for evaluating the solution, based on up-sampling and the boundary element method, that is efficient and accurate throughout the domain, circumventing the close-evaluation problem. To support the design choices of the numerical method, we derive regularity estimates with bounds explicitly in terms of the conductivities and the geometries of the boundaries between their regions. The resulting method is fast and accurate for solving for the electrostatic potential in media with piecewise constant conductivities. We also provide analytical results for the system of equations for the charge densities. Firstly, we establish existence and uniqueness to this system of equations. Secondly, we derive regularity for the charge densities along each interface. We show that assuming that the interface has $\mathscr{C}^k$ regularity, then the charge density is of regularity $H^k$ (*i.e.*, in the Hilbert space of order $k$). Furthermore, we generalize our results by considering the case where the piecewise constant regions of conductivity overlap and we study the behaviour of the solution to leading order at points of intersection between two transversely intersecting interfaces of regions of piecewise constant conductivity.