UNIVERSITY OF TORONTO
FACULTY OF ARTS & SCIENCE | Department of Mathematics

Departmental PhD Thesis Exam

Tuesday, September 10th, 2024 at 1:00 p.m. (sharp)
via Zoom / BA6183

PhD Candidate : Lemonte Alie-Lamarche

Supervisor : Bálint Virág

Thesis title : The Hausdorff Dimension of the Level Sets of the Directed Landscape

Abstract

The directed landscape $\mathscr{L}$ introduced by Dauvergne, Ortmann, and Virág in 2018 has rapidly become a central object of study in modern probability theory. It is believed by many that this random directed metric is possibly *the* universal scaling limit of the random growth models in the KPZ universality class. It was proven by Dauvergne, Nica, and Virág in 2021 in that the directed landscape is, among other things, the scaling limit of at least six different models of last passage percolation in the uniform on compact topology. This universality of $\mathscr{L}$ as a scaling limit, as well as its ties to other random growth models, makes understanding anything about its fractal structure and geometry of significant interest in the wider long-term endeavour to fully understand the structure of the KPZ universality class.

In this thesis, we prove several results about the fractal structure of the level sets of $\mathscr{L}(0,0;\cdot,\cdot)$ as a function on $\mathbb{R}\times\mathbb{R}_{>0}$, which translate quite easily into very similar statements about the corresponding level sets of $\mathscr{L}$ on its domain. We first prove that the $h-$level sets of rescaled Exponential last passage percolation starting at (0,0) converge in the Hausdorff metric induced by the Euclidean norm to the $h-$level set of $\mathscr{L}(0,0;\cdot,\cdot)$ on any convex compact set $K\subseteq\mathbb{R}\times\mathbb{R}_{>0.}$ We then prove that the Hausdorff dimension of the $h-$level set of $\mathscr{L}(0,0;\cdot,\cdot)$ is at most $\frac{5}{3}$ almost surely for all $h\in\mathbb{R}$. We conclude this thesis by developing a strategy to systematically find lower bounds on the Hausdorff dimension of random $h-$level sets of stochastic processes indexed by $\mathbb{R}^2$ that hold with a positive $h-$dependent probability p_h. We apply this strategy to $\mathscr{L}(0,0;\cdot,\cdot)$ to establish that the $h-$ level set of $\mathscr{L}(0,0;\cdot,\cdot)$ has Hausdorff dimension at least $\frac{3}{2}$ with a positive $h-$dependent probability. In the process of doing so, we also construct a partial-two point bound for the function $\mathscr{L}(0,0;\cdot,\cdot)$. This thesis is based on joint work conducted with Virginia Pedreira under the supervision of Bálint Virág.